Some Applications of String Field Theory

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Plan

1. Review of divergences in string theory

2. Use of string field theory in removing divergences.

3. Structure of string field theory

4. More applications

Superstring $\equiv$ heterotic or type II strings

(includes compactified theories with non-trivial NS background)

In string theory the observables are S-matrix elements.

The prescription for computing S-matrix is apparently different from that in quantum field theories.

g-loop, N-point amplitude:

$$\int dm_1 \cdots dm_{6g-6+2N}\, F(m_1, \cdots m_{6g-6+2N})$$

$\{m_i\}$ **parametrize moduli space of two dimensional Riemann surfaces of genus g and N marked points.**

$F(\{m_i\})$**: some correlation function of a two dimensional conformal field theory on the Riemann surface.**

The closest comparison between string theory amplitudes and field theory amplitudes can be made in Schwinger parameter representation of the latter

$$(k^2 + m^2)^{-1} = \int_0^\infty ds\, e^{-s(k^2+m^2)}$$

1. Replace each propagator by this in the Feynman amplitude.

2. Carry out the loop momentum integrals since they are gaussian integrals.

Result

$$\int ds_1 \cdots ds_n f(s_1, \cdots s_n)$$

m_i's resemble s_i's and F resembles f.

Field theory (contd)

Ultraviolet (UV) divergence $\Leftrightarrow$ large loop momentum.

Infrared (IR) divergence $\Leftrightarrow$ vanishing $k^2 + m^2$.

$$(k^2 + m^2)^{-1} = \int_0^\infty ds\, e^{-s(k^2+m^2)}$$

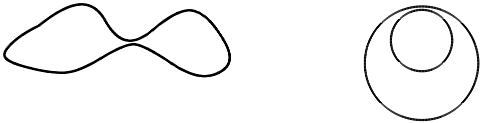
After integration over the momenta, we cannot classify UV and IR divergences as coming from large and small momenta.

Dictionary:

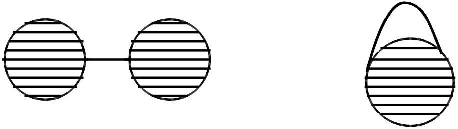
1. UV divergences come from $s \to 0$

2. IR divergences come from $s \to \infty$

Divergences in string theory are associated with degenerate Riemann surfaces



Resembles Feynman diagrams with a large s propagator



Under $\{m_i\} \Leftrightarrow \{s_j\}$ identification, the size of the small cycle near degeneration goes as e^{-s}.

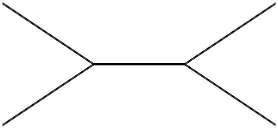
Degeneration $\Rightarrow s \to \infty \Rightarrow$ IR divergence

This shows that string theory is free from ultraviolet divergences but suffers from infrared divergences

Sources of infrared divergence in string theory can be understood from the divergences in field theory amplitudes in large s limit

$$(k^2 + m^2)^{-1} = \int_0^\infty ds\, e^{-s(k^2+m^2)}$$

1. For $k^2 + m^2 < 0$, l.h.s. is finite but r.h.s. diverges



– can be dealt with in quantum field theory by working directly with l.h.s.

– in conventional string perturbation theory these divergences have to be circumvented via analytic continuation / deformation of moduli space integration contours

D'Hoker, Phong; Berera; Witten; $\cdots$

$$(k^2 + m^2)^{-1} = \int_0^\infty ds\, e^{-s(k^2+m^2)}$$

2. For $(k^2 + m^2) = 0$, l.h.s. and r.h.s. both diverge.

– present in quantum field theories e.g. in external state mass renormalization and massless tadpole diagrams

– have to be dealt with using renormalized mass and correct vacuum.

In standard superstring perturbation theory these divergences have no remedy.

Superstring field theory is a quantum field theory whose amplitudes, computed with Feynman diagrams, should have the following properties:

1. They agree with standard superstring amplitudes when the latter are finite

2. They agree with <u>analytic continuation</u> of standard superstring amplitudes when the latter are finite

3. They formally agree with standard superstring amplitudes when the latter have genuine divergences, but $\cdots$

$\cdots$ in superstring field theory we can deal with these divergences using standard field theory techniques like mass renormalization and shift of vacuum.

Does such a theory exist?

For bosnic strings, there is such a field theory Zwiebach

For superstrings there is an apparent no go theorem.

If we can construct an action for type IIB superstring theory then by taking its low energy limit we should get an action for type IIB supergravity

– known to be not possible due to the existence of a 4-form field with self-dual field strength.

Therefore construction of an action for type IIB superstring theory should be impossible.

Resolution

It is possible to construct actions for heterotic and type II string field theory, but the theory contains an additional set of particles which are free.

These additional particles are unobservable since they do not scatter.

Note: For classical open superstring field theory there are other approaches.

Kunitomo, Okawa; Erler, Konopka, Sachs; Erler, Okawa, Takazaki; Konopka, Sachs; $\cdots$

Structure of the action

Two sets of string fields, ψ and ϕ

Each is an infinite component field, represented as a vector

Action takes the form

$$\mathbf{S} = \left[-\frac{\mathbf{1}}{\mathbf{2}}(\phi, \mathbf{QX}\phi) + (\phi, \mathbf{Q}\psi) + \mathbf{f}(\psi) \right]$$

Q, X: commuting linear operators

(,): Lorentz invariant inner product

f(ψ): a functional of ψ describing interaction term.

Some technical details (for heterotic string)

$$S = \left[-\frac{1}{2}(\phi, QX\phi) + (\phi, Q\psi) + f(\psi)\right]$$

ψ has picture numbers $(-1, -1/2)$ in (NS,R) sectors

ϕ has picture numbers $(-1, -3/2)$ in (NS,R) sector

Q: BRST operator

X: (Identity, zero mode of PCO) in the (NS, R) sectors.

f(ψ): given by an integral over subspace of moduli space of Riemann surfaces

Integrand: correlation function of ψ states, PCO's, ghosts etc.

The subspace never includes degenerate Riemann surfaces.

$$\mathbf{S} = \left[-\frac{1}{2}(\phi, \mathbf{Q}\,\mathbf{X}\,\phi) + (\phi, \mathbf{Q}\,\psi) + \mathbf{f}(\psi) \right]$$

Equations of motion:

$$\mathbf{Q}(\psi - \mathbf{X}\,\phi) = \mathbf{0}$$

$$\mathbf{Q}\phi + \mathbf{f}'(\psi) = \mathbf{0}$$

first + X $\times$ second equation gives

$$\mathbf{Q}\psi + \mathbf{X}\,\mathbf{f}'(\psi) = \mathbf{0}$$

ψ **describes interacting fields**

Rest of the independent degrees of freedom describe decoupled free fields.

This action has infinite dimensional gauge invariance

– can be quantized using Batalin-Vilkovisky formalism

1. Gauge fix

2. Derive Feynman rules

3. Compute amplitudes

If we use Schwinger parameter representation of the propagators then the Feynman amplitudes for ψ reproduce the usual string theory amplitudes.

Each diagram represents integration over part of the moduli space of Riemann surfaces.

Sum of all diagrams gives integration over the full moduli space.

Rest of the degrees of freedom decouple and will be irrelevant for our analysis.

Such an amplitude will have the usual divergences of string perturbation theory.

All such divergences now come in the limit when one or more Scwinger parameters become large

– usual IR divergences in quantum field theories

However since we have an underlying field theory, we can deal with these divergences following the usual procedure of a field theory.

1. Find 1PI effective action.

2. Find the extremum of this action.

3. Find solutions of linearized equations of motion around the extremum to find renormalized masses.

4. Compute S-matrix using LSZ formalism.

More details on the amplitudes

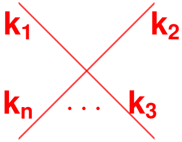
The tree level propagators have standard form in the 'Siegel gauge'

$$(\mathbf{L_0} + \bar{\mathbf{L}}_0)^{-1}\, \mathbf{X}\, \mathbf{b_0}\, \bar{\mathbf{b}}_0\, \delta_{\mathbf{L_0},\bar{\mathbf{L}}_0}$$

In momentum space

$$(\mathbf{k}^2 + \mathbf{M}^2)^{-1} \times \textbf{polynomial in momentum}$$

The polynomial comes from matrix element of $\mathbf{X}\, \mathbf{b_0}\, \bar{\mathbf{b}}_0$.



Vertices are accompanied by a suppression factor of

$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right]$$

A: a positive constant that can be made large by a non-linear field redefinition (adding stubs). Hata, Zwiebach

This makes

– momentum integrals UV finite (almost)

– sum over intermediate states converge

Momentum dependence of vertex includes

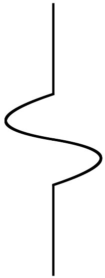
$$\exp\left[-\frac{A}{2}\sum_i (k_i^2 + m_i^2)\right] = \exp\left[-\frac{A}{2}\sum_i (\vec{k}_i^{\,2} + m_i^2) + \frac{A}{2}(k_i^0)^2\right]$$

Integration over $\vec{k}_i$ is convergent for large $\vec{k}_i$, but integration over k_i^0 diverges at large k_i^0.

The spatial components of loop momenta can be integrated along the real axis, but we have to treat integration over loop energies more carefully.

Resolution: Need to have the ends of loop energy integrals approach $\pm i\infty$.

In the interior the contour has to be deformed away from the imaginary axis to avoid poles from the propagators.



We shall now describe in detail how to choose the loop energy integration contour.

General procedure:

Pius, A.S.

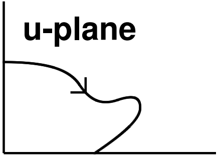
1. Multiply all external energies by a complex number u.

2. For u=i, all external energies are imaginary, and we can take all loop energy contours to lie along the imaginary axis without encountering any singularity.

3. Now deform u to 1 along the first quadrant.

4. If some pole of a propagator approaches the loop energy integration contours, deform the contours away from the poles, keeping their ends at $\pm i\infty$.

Complex u-plane



With this definition the amplitude develops an imaginary part which is normally absent in superstring perturbation theory before analytic continuation.

Result 1: Such deformations are always possible as long as u lies in the first quadrant

– the loop energy contours do not get pinched by poles approaching each other from opposite sides.

Result 2: The amplitudes computed this way satisfy Cutkosky cutting rules

Pius, A.S.

– relates $T - T^\dagger$ to $T^\dagger T$ $\quad\quad$ $S = 1 - i\,T$

– proved by using contour deformation in complex loop energy plane

This is a step towards proof of unitarity but not a complete proof

In $T^\dagger T = T^\dagger |n\rangle\langle n| T$, the sum over intermediate states runs over all states in Siegel gauge.

Desired result: Only physical states should contribute to the sum.

This can be proved using the quantum Ward identities of superstring field theory A.S.

– requires cancellation between matter and ghost loops

The proof of unitarity takes into account

1. Mass and wave-function renormalization effects and lifting of degeneracy

2. The fact that some (most) of the string states become unstable under quantum corrections.

3. The possible shift in the vacuum due to quantum effects.

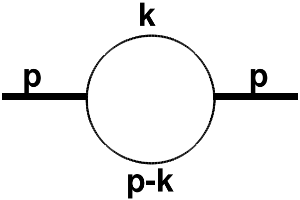
It does not take into account the infrared divergences from soft particles arising in D $\leq$ 4.

(String field theory version of Kinoshita, Lee, Nauenberg theorem has not yet been proven.)

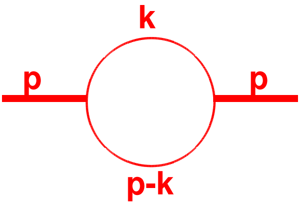
An example:

Consider two fields, one of mass M and another of mass m, with $M>2m$.

Consider one loop mass renormalization of the heavy particle.



Thick line: heavy particle   Thin line: light particle.



$$\delta M^2 = i \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2 + m^2\} - A\{(p-k)^2 + m^2\}] \{k^2 + m^2\}^{-1}\{(p-k)^2 + m^2\}^{-1} B(k)$$

B(k): a polynomial in momentum encoding additional contribution to the vertices and / or propagators.

We shall work in $\vec{p} = 0$ frame, and take $p^0 \to M$ limit from the first quadrant.

$$\delta M^2 = i \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2 + m^2\} - A\{(p-k)^2 + m^2\}]$$
$$\{k^2 + m^2\}^{-1}\{(p-k)^2 + m^2\}^{-1} B(k)$$

Poles in the k^0 plane (for $\vec{p} = 0$):

$$Q_1 \equiv \sqrt{\vec{k}^2 + m^2}, \quad Q_2 \equiv -\sqrt{\vec{k}^2 + m^2},$$

$$Q_3 \equiv p^0 + \sqrt{\vec{k}^2 + m^2}, \quad Q_4 \equiv p^0 - \sqrt{\vec{k}^2 + m^2}$$

For p^0 imaginary, take k^0 contour along imaginary axis.

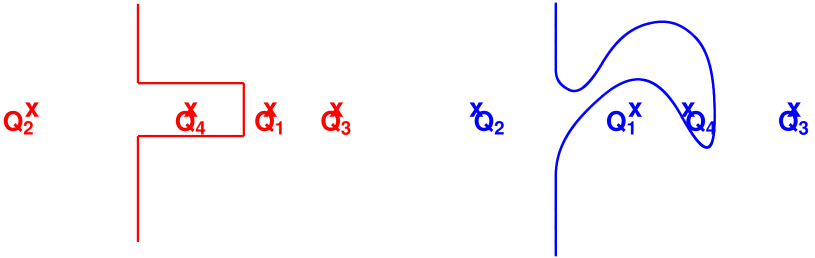
Q_1, Q_3 to the right and Q_2, Q_4 to the left of the imaginary axis.

$$Q_1 \equiv \sqrt{\vec{k}^2 + m^2}, \quad Q_2 \equiv -\sqrt{\vec{k}^2 + m^2},$$

$$Q_3 \equiv p^0 + \sqrt{\vec{k}^2 + m^2}, \quad Q_4 \equiv p^0 - \sqrt{\vec{k}^2 + m^2}$$

As p^0 approaches real axis, the poles approach the real axis.

Two situations depending on the value of $\vec{k}$.



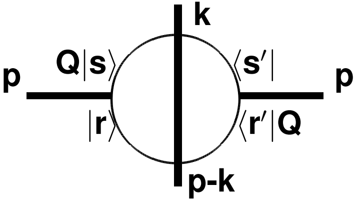
Note: Q_1, Q_3 to the right and Q_2, Q_4 to the left of the contour in both diagrams.

Complex conjugate contours giving $(\delta M^2)^*$

– can be deformed to each other without picking any residue unless $Q_4 \to Q_1$ putting both lines on-shell.

Residue given by Cutkosky rules.

The cut diagrams in string field theory will have some unwanted terms



These two diagrams cancel using Ward identity.

All order proof of unitarity involves generalization of this type of analysis

– takes into account quantum modification of the BRST operator Q computed from 1PI effective action.

String motivated approach: Evaluate the original integral using Schwinger parametrization

$$\exp[-A(k^2+m^2)](k^2+m^2)^{-1} = \int_A^\infty dt_1 \exp[-t_1(k^2+m^2)]$$

$$\exp[-A((p-k)^2+m^2)]((p-k)^2+m^2)^{-1} = \int_A^\infty dt_2 \exp[-t_2((p-k)^2+m^2)]$$

For constant B, after doing momentum integrals (formally)

$$\delta M^2 = -B\,(4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2\,(t_1+t_2)^{-D/2} \exp\left[\frac{t_1 t_2}{t_1+t_2} M^2 - (t_1+t_2)m^2\right]$$

– diverges from the upper end for $M > 2m$.

– can be traced to the impossibility of choosing energy integration contour keeping Re(k^2+m^2)>0, Re($(p-k)^2+m^2$)>0.

$$\mathrm{i\,B} \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2+m^2\} - A\{(p-k)^2+m^2\}]$$
$$\{k^2+m^2\}^{-1}\{(p-k)^2+m^2\}^{-1}$$ **finite**

$$\text{‘=’} \quad -B\,(4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2\,(t_1+t_2)^{-D/2}$$
$$\exp\left[\frac{t_1 t_2}{t_1+t_2} M^2 - (t_1+t_2)m^2\right]$$ **divergent**

More generally for a polynomial B, we have a polynomial P s.t.

$$\mathrm{i} \int \frac{d^D k}{(2\pi)^D} \exp[-A\{k^2+m^2\} - A\{(p-k)^2+m^2\}]$$
$$\{k^2+m^2\}^{-1}\{(p-k)^2+m^2\}^{-1}\,B(k)$$

$$\text{‘=’} \quad -(4\pi)^{-D/2} \int_A^\infty dt_1 \int_A^\infty dt_2\,(t_1+t_2)^{-D/2}$$
$$\exp\left[\frac{t_1 t_2}{t_1+t_2} M^2 - (t_1+t_2)m^2\right] P(1/(t_1+t_2), t_2/(t_1+t_2))$$

Such divergences arise in actual computation of one loop two point functions in heterotic and type II string theories.

Using these ‘identities’ we can convert these divergent expressions into finite expressions

– have both real and imaginary parts consistent with unitarity.

An alternative strategy

Berera; Witten

Turn the upper limits of the t_i integrals towards $i\infty$ instead of ∞

$$-(4\pi)^{-D/2} \int_A^{\infty} dt_1 \int_A^{\infty} dt_2 \, (t_1 + t_2)^{-D/2}$$
$$\exp\left[\frac{t_1 t_2}{t_1 + t_2} M^2 - (t_1 + t_2) m^2\right] P(1/(t_1 + t_2), t_2/(t_1 + t_2))$$

becomes finite with this prescription for $D > 4$.

At one loop this prescription agrees with the intergration rules over loop energies.

The status at higher loops is not clear yet.

A specific example in string theory

One loop mass renormalization of the lowest massive state on the leading Regge trajectory in the heterotic string theory

Need to compute torus two point function of on-shell states

On shell two point function gives

$$\delta M^2 = -\frac{1}{32\,\pi} M^2 g^2 \int d^2\tau \int d^2 z \; F(z, \bar{z}, \tau, \bar{\tau}),$$

$$F(z, \bar{z}, \tau, \bar{\tau}) \equiv \left\{ \sum_\nu \overline{\vartheta_\nu(0)^{16}} \right\} (\overline{\eta(\tau)})^{-18} (\eta(\tau))^{-6} (\overline{\vartheta_1'(0)})^{-4} \left(\vartheta_1(z) \overline{\vartheta_1(z)} \right)^2$$

$$\left[\left(\frac{\overline{\vartheta_1'(z)}}{\overline{\vartheta_1(z)}} \right)^2 - \frac{\overline{\vartheta_1''(z)}}{\overline{\vartheta_1(z)}} - \frac{\pi}{\tau_2} \right]^2 \exp[-4\pi\, z_2^2/\tau_2]\, (\tau_2)^{-5},$$

$z = z_1 + i\, z_2 \in$ **torus,** $\tau = \tau_1 + i\tau_2 \in$ **fundamental region**

$\vartheta_1, \cdots \vartheta_4$**: Jacobi theta functions** η**: Dedekind function**

For large z_2 and $\tau_2 - z_2$, F has a growing part

$$2\,(2\pi)^{-4}\,\left(32\pi^4 - 32\frac{\pi^3}{\tau_2} + 512\,\frac{\pi^2}{\tau_2^2}\right)\,\exp[4\pi z_2 - 4\pi z_2^2/\tau_2]\,\tau_2^{-5}$$

$\Rightarrow$ divergent integral.

Divergent part after using $t_1 = \pi z_2, \quad t_2 = \pi(\tau_2 - z_2)$

$$J = -2^{-3}\pi^2\,M^2\int_A^\infty dt_1\int_A^\infty dt_2\,(t_1+t_2)^{-5}$$
$$\left(1 - \frac{1}{(t_1+t_2)} + 16\,\frac{1}{(t_1+t_2)^2}\right)\exp\left[4\frac{t_1 t_2}{t_1+t_2}\right]$$

A: arbitrary constant

J is divergent, but the integral matches the one we analyzed before for field theory with m=0, M=2

Strategy (can be justified using string field theory):

Use the previous ‘identities’ to replace J by the momentum space integral

$$J = i\,(2\pi)^7 M^2 g^2 \int \frac{d^{10}k}{(2\pi)^{10}} \exp[-Ak^2 - A(p-k)^2]$$
$$(k^2)^{-1}\,\{(p-k)^2\}^{-1}\,\{1 - 2\,(k^1)^2 + 64\,(k^1)^2\,(k^2)^2\}$$

– finite integral once we choose the integration contour for energy integral following the procedure described earlier.

To this we add the finite part which is given by usual integral over moduli space with the divergent part <u>subtracted</u>.

Final result gives finite real and imaginary parts in accordance with unitarity.

Summary

Covariant superstring field theory gives a Lorentz invariant, ultraviolet finite and unitary theory.

Divergences associated with mass renormalization and shift of vacuum can be dealt with as in conventional quantum field theories.

It can also provide useful alternative to analytic continuation that is often needed in conventional superstring perturbation theory to make sense of divergent results.